New coloring applications of the Regularity Lemma

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We present some new coloring applications of the Regularity Lemma. Among other results, by proving an old conjecture of Faudree and Schelp, we determine the exact three color Ramsey numbers for paths. More precisely we show that $R(P_n, P_n, P_n)$ is $2n-1$ if n is odd, and $2n-2$ if n is even.

We also study Ramsey numbers for Berge hypergraph cycles, where we define a Berge-cycle in the following way. Let $\mathcal{H}$ be an r-uniform hypergraph (a family of some r-element subsets of a set). For vertices $x, y \in V(\mathcal{H})$ we say x is adjacent to y, if there exists an edge $e \in E(\mathcal{H})$ such that $x, y \in e$. An r-uniform ℓ-cycle, or *Berge-cycle* of length ℓ, denoted $C_\ell^{(r)}$, is a sequence of distinct vertices $v_1, v_2, \ldots, v_\ell$, the *core of the cycle* , such that each v_i is adjacent to v_{i+1} and the edges e_i that contain v_i, v_{i+1} are all distinct for $i, 1 \leq i \leq \ell$ where $v_{\ell+1} \equiv v_1$. We pay special attention to the case when we can guarantee a monochromatic *Hamiltonian* Berge-cycle.

Co-authors on this work included András Gyárfás, Jenő Lehel, Miklós Ruszinkó, Richard Schelp and Endre Szemerédi.