Degree bounded factorizations of bipartite multigraphs and of pseudographs

A.J.W. Hilton
(Reading University and Queen Mary University of London)

For $d \geq 1,\ s \geq 0$ a $(d, d+s)$-*graph* is a graph whose degrees all lie in the interval $\{d, d+1, \dots, d+s\}$. For $r \geq 1,\ a \geq 0$ an $(r, r+a)$-*factor* of a graph G is a spanning $(r, r+a)$-subgraph of G. An $(r, r+a)$-*factorization* of a graph G is a decomposition of G into edge-disjoint $(r, r+a)$-factors.

We prove a number of results about $(r, r+a)$-factorizations of $(d, d+s)$-bipartite multigraphs and of $(d, d+s)$-pseudographs (multigraphs with loops permitted). For example, for $t \geq 1$ let $\beta(r, s, a, t)$ be the least integer such that, if $d \geq \beta(r, s, a, t)$ then every $(d, d+s)$-bipartite multigraph G has an $(r, r+a)$-factorization into x $(r, r+a)$-factors for at least t different values of x. Then we show that

$$\beta(r, s, a, t) = r \left\lceil \frac{tr + s - 1}{a} \right\rceil + (t-1)r \,.$$

Similarly for $t \geq 1$ let $\pi(r, s, a, t)$ be the least integer such that if $d \geq \pi(r, s, a, t)$ then each $(d, d+s)$-pseudograph has an $(r, r+a)$-factorization into x $(r, r+a)$-factors for at least t different values of x. We show that, if r and a are even, then $\pi(r, s, a, t)$ is given by the same formula.

We use this to give bounds for $\pi(r, s, a, t)$ when r and a are not both even. Finally we consider the corresponding functions for multigraphs without loops, and for simple graphs.