Hall t–chromatic graphs

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Suppose that G is a simple graph, L is an assignment of finite sets to the vertices of G, and K is an assignment of non-negative integers to those vertices. A proper (L, K)–coloring of G is a selection of subsets of the sets $L(v)$, with each such subset having $K(v)$ elements, such that the subsets selected for adjacent vertices are disjoint.

Hall's condition on G, L, and K is an elementary necessary condition for the existence of a proper (L, K)–coloring of G; the condition descends from the famous theorem of Philip Hall on systems of distinct representatives – hence the name. A graph G is Hall t–chromatic if Hall's condition suffices for a proper coloring whenever L is a constant assignment of a set with t elements.

So far the study of these properties has yielded some pretty results, some easy indirect proofs that the fractional chromatic numbers of certain graphs are equal to their Hall ratios (usually a chore to verify, when true), and a bushel of open questions.